CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination, $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

   (a) On what intervals is f increasing?

   (b) On what intervals is the graph of f concave upward?

   (c) Write the equation of each horizontal tangent line to the graph of f.

2. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t - 1)(t - 3)$. At time $t = 2$, the position of the particle is $x(2) = 0$.

   (a) Find the minimum acceleration of the particle.

   (b) Find the total distance traveled by the particle.

   (c) Find the average velocity of the particle over the interval $0 \le t \le 5$.

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3. Let f be the function given by $f(x) = \ln \left| \dfrac{x}{1+x^2} \right|$.

    (a) Find the domain of f.

    (b) Determine whether f is an even function, an odd function, or neither. Justify your conclusion.

    (c) At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether $f(x)$ is a relative maximum or a relative minimum.

    (d) Find the range of f.

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4. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

    (a) Find $\dfrac{dy}{dx}$ in terms of y. _1 - Siny_

    (b) Write an equation for each vertical tangent to the curve. _Why not_ _$4 - \frac{\pi}{2} = ?(2 - ?)$_

    (c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

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5. Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

   (a) Find the x-coordinate of the point of tangency and the value of k.

   (b) Let R be the region enclosed by the y-axis and the graphs of f and g. Using the results found in part (a), determine the area of R.

   (c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part (b), about the x-axis.

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6. At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with radius r is $V = \frac{4}{3} \pi r^3$.)

   (a) Find the radius of the sphere as a function of t.

   (b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

END OF EXAMINATION.